Isentropic Thinking
&
Airstreams Analysis

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Utility of Isentropic Analysis

- Diagnose and visualize vertical motion - through advection of pressure and system relative flow
- Depict 3-Dimensional advection of moisture
- Compute moisture stability flux - dynamic destabilization and moistening of environment
- Diagnose isentropic potential vorticity
- Diagnose dry static stability (plan or cross-section view) and upper-level frontal zones
- Diagnose conditional symmetric instability (slantwise convection)
- Help depict 2-D frontogenetical and transverse jet streak circulations on cross sections
Theta as a Vertical Coordinate

- \( \theta = T \left( \frac{1000}{P} \right)^\kappa \), where \( \kappa = \frac{R_d}{C_p} \)
- Entropy = \( \Phi = C_p \ln \theta + \text{const} \)
- If \( \Phi = \text{const} \) then \( \theta = \text{const} \), so constant entropy sfc = isentropic sfc
- Three types of stability, since \( \frac{\partial \theta}{\partial z} = \left( \frac{\theta}{T} \right) \left[ \Gamma_d - \gamma \right] \)
  - **stable**: \( \gamma < \Gamma_d \), \( \theta \) increases with height
  - **neutral**: \( \gamma = \Gamma_d \), \( \theta \) is constant with height
  - **unstable**: \( \gamma > \Gamma_d \), \( \theta \) decreases with height
- So, isentropic surfaces are closer together in the vertical in stable air and further apart in less stable air
Visualizing Static Stability – Vertical Gradients of $\theta$

Vertical changes of potential temperature related to lapse rates:

U = unstable
N = neutral
S = stable
VS = very stable
Theta as a Vertical Coordinate

• Isentropes slope **DOWN** toward warm air, **UP** toward cold air – this is opposite to the slope of pressure surfaces: since $\theta = T (1000/P)^k$,
as $P$ increases (decreases), $T$ increases (decreases) to keep $\theta$ constant (as on a skew-T diagram).

• Isentropes slope much greater than pressure surfaces given the same thermal gradient; as much as one order of magnitude more!

• On an isentropic surface an isotherm = an isobar = an isopycnic (const density); (remember: $P = \rho R_d T$)

• On an isentropic surface we analyze the Montgomery streamfunction to depict geostrophic flow, where:

\[
M = \Psi = C_p T + gZ ; \quad \vec{V}_{geo} = \frac{\hat{k}}{f} \times \nabla \psi
\]
Isentropic Analysis: Advantages

- For synoptic scale motions, in the absence of diabatic processes, isentropic surfaces are material surfaces, i.e., parcels are thermodynamical bound to the surface.
- Horizontal flow along an isentropic surface contains the adiabatic component of vertical motion often neglected in a Z or P reference system.
- Moisture transport on an isentropic surface is three-dimensional - patterns are more spatially and temporally coherent than on pressure surfaces.
- Isentropic surfaces tend to run parallel to frontal zones making the variation of basic quantities (u,v, T, q) more gradual along them.
Plan view of an isentropic surface analysis

Blue dashed lines = isobars, red dashed lines are isohumes
Moisture transport and lift along an isentropic surface
Three-Dimensional Isentropic Topography
Moist air from low levels on the left (south) is transported upward and to the right (north) along the isentropic surface. However, in pressure coordinates water vapor appears on the constant pressure surface labeled $p$ in the absence of advection along the pressure surface -- it appears to come from nowhere as it emerges from another pressure surface. (adapted from Bluestein, vol. I, 1992, p. 23)
Relative Humidity
305K surface
12 UTC 3-17-87
RH>80% = green

Pressure analysis
305K surface
12 UTC 3-17-87

Benjamin et al.
Relative Humidity at 500 hPa
RH > 70% = green
Isentropic Analysis: Advantages

- Atmospheric variables tend to be better correlated along an isentropic surface upstream/downstream, than on a constant pressure surface, especially in advective flow.
- The vertical spacing between isentropic surfaces is a measure of the dry static stability. Convergence (divergence) between two isentropic surfaces decreases (increases) the static stability in the layer.
- The slope of an isentropic surface (or pressure gradient along it) is directly related to the thermal wind.
- Parcel trajectories can easily be computed on an isentropic surface. Lagrangian (parcel) vertical motion fields are better correlated to satellite imagery than Eulerian (instantaneous) vertical motion fields.
Figure 4. Schematic depiction of isentropic surfaces (dashed lines) and representative pressure surface (solid line) in the vicinity of a cold frontal zone (bold, curved lines).
Surface Map for 00 UTC 30 December 1990
Surface Map for 12 UTC 30 December 1990
Sounding for Paducah, KY 30 December 1990 12 UTC

898 hPa
+14.0 C

962 hPa
-1.3C
Cross Section Taken Normal to Arctic Frontal Zone: 12 UTC 30 December 1990
Cross section for 00 UTC 17 April 1976 from Winslow, AZ to Tucson, AZ to Fraccionamiento, MX. Solid lines are isentropes and dashed lines are isotachs (m s\(^{-1}\)). (Shapiro 1981, JAS)

Note how the isentropes run parallel to the frontal zone – NOT across it; also note the strong vertical wind shear
Station Model for Isentropic Analysis

Pressure (mb)

Montgomery Streamfunction (J kg\(^{-1}\) or m\(^2\) s\(^{-2}\))

Mixing Ratio (gm kg\(^{-1}\))

Wind Direction and Speed (knots)

Static Stability over 8 K layer (4 K below and 4 K above) in mb

785

29460

10

85
Choosing the “Right” Isentropic Surface(s)

- The “best” isentropic surface to diagnose low-level moisture and vertical motion varies with latitude, season, and the synoptic situation. There are various approaches to choosing the “best” surface(s):
- Use the ranges suggested by Namias (1940):
  - **Season** | **Low-Level Isentropic Surface**
    - Winter    | 290-295 K
    - Spring    | 295-300 K
    - Summer    | 310-315 K
    - Fall      | 300-305 K
Choosing the “Right” Isentropic Surface(s)

BEST METHOD:

- Compute a cross section of isentropes and isohumes (mixing ratios) normal to a jet streak or baroclinic zone in the area of interest.
- Choose the low-level isentropic surface that is in the moist layer, displays the greatest slope, and stays 50-100 hPa above the surface.
- A rule of thumb is to choose an isentropic surface that is located at ~700-750 hPa above your area (as with the “Garcia” method for forecasting snowfall)
Using an Isentropic Cross Section to Choose a $\theta$ Surface:
Isentropic Cross Section for 00 UTC 05 Dec 1999

E488: Lat/Lon 44N/100W => 32N/93W :FHR= 0 :FHR= 0/24 :FIL= de059900.e48
99/12/05/00--MIXR C1-3 DASH CLR5&
Isentropic Moisture Parameters

• **Lifting Condensation Pressure (LCP):** The pressure to which a parcel of air must be raised dry-adiabatically in order to reach condensation. Represents moisture differences better than mixing ratio at low values of mixing ratio. Condensation pressure on an isentropic surface is equivalent to dew point on a constant pressure surface.

• **Condensation Difference (CD):** The difference between the actual pressure and the condensation pressure for a point on a isentropic surface. The smaller the condensation difference, the closer the point is to saturation. Due to smoothing and round off errors, a difference < 20 hPa represents saturation. Values < 100 hPa indicated near saturation. Condensation difference on an isentropic surface is equivalent to dew point depression on a constant pressure surface.
Isentropic Moisture Parameters

Moisture Transport Vectors (MTV):

• Defined as the product of the horizontal velocity vector, $V$, and the mixing ratio, $q$. Units are $\text{gm-m/kg-s}$; values typically range from 50-250, depending upon the level and the season.

• Typically, stable precipitation due to isentropic upglide falls downstream from the maximum of the moisture transport vector magnitude in the northern gradient region. The moisture transport vectors and isopleths of the magnitude of the moisture transport vectors are usually displayed.

• Note that the negative divergence of the MTVs is equal to the horizontal moisture convergence, since

$$MCON = -\nabla \cdot \vec{V}q$$

where: $\vec{V}q = MTV$
Isentropic Analysis: Disadvantages

• The temporal continuity of isentropic surfaces are disrupted by diabatic heating/cooling; diabatic heating (cooling) causes isentropic surfaces to move downward (upward)

• Isentropic surfaces are vertical in regions of neutral lapse rates (i.e., $\gamma = \Gamma_d$) and “fold” in regions of superadiabatic lapse rates (i.e., $\gamma > \Gamma_d$)

• Forecaster must choose 1-2 isentropic surfaces to analyze in lower troposphere, not pre-selected like pressure surfaces.
  • Construct an isentropic cross section along with isohumues normal to the baroclinic zone and choose an isentropic surface – but stay out of that nasty boundary layer!
Neutral-Superadiabatic Lapse Rates

SA = superadiabatic and N = neutral
Vertical Resolution is a Function of Static Stability

LS = less stable (weak static stability) and VS = very stable (strong static stability)
Radiational Heating/Cooling Disrupts the Continuity of Isentropic Surfaces

As time increases solar heating causes the 300 K isentropic surface to become “redefined” at higher pressures

Namias, 1940: An Introduction to the Study of Air Mass and Isentropic Analysis, AMS, Boston, MA.
304 K Isentropic Surface for 12 UTC 2 May 2002
304 K Isentropic Surface for 00 UTC 3 May 2002

Note loss of data in SE U.S. and in Texas...304 K surface went underground
Computing Vertical Motion

\[ \omega = \frac{dp}{dt} = \left( \frac{\partial P}{\partial t} \right) + \mathbf{V} \cdot \nabla P + \frac{d \theta}{dt} \frac{\partial P}{\partial \theta} \]

**Term A:** local pressure change on the isentropic surface

**Term B:** advection of pressure on the isentropic surface

**Term C:** diabatic heating/cooling term (modulated by the dry static stability.

Typically, at the synoptic scale it is assumed that terms A and C are nearly equal in magnitude and opposite in sign.
Example of Computing Vertical Motion

1. Assume isentropic surface descends as it is warmed by latent heating (local pressure tendency term):

\[ \frac{\partial P}{\partial t} = 650 - 550 \text{ hPa} / 12 \text{ h} = +2.3 \mu\text{bars s}^{-1} \text{ (descent)} \]

2. Assume 50 knot wind is blowing normal to the isobars from high to low pressure (advection term):

\[ V \cdot \nabla P = (25 \text{ m s}^{-1}) \times (50 \text{ hPa/300 km}) \times \cos 180^\circ \]

\[ V \cdot \nabla P = -4.2 \mu\text{bars s}^{-1} \text{ (ascent)} \]

3. Assume 7 K diabatic heating in 12 h in a layer where \( \theta \) increases 4 K over 50 hPa (diabatic heating/cooling term):

\[ \left( \frac{d\theta}{dt} \right)(\frac{\partial P}{\partial \theta}) = (7 \text{ K/12 h})(-50 \text{ hPa/4K}) = -2 \mu\text{bars s}^{-1} \]

\( \text{(ascent)} \)
Local pressure tendency term computed over 12, 6 and 3 hours by Homan and Uccellini, 1987 (WAF, vol. 2, 206-228)

Local pressure tendencies should be computed over 3 h time periods for most accurate results.

<table>
<thead>
<tr>
<th>12-h p Tendency and $\omega$ on 288 K Surface</th>
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</thead>
<tbody>
<tr>
<td>1200 UTC 22 February to 0000 UTC 23 February</td>
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<tr>
<td>Tendency (mb/12 h)</td>
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<td>IAD</td>
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<table>
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<td>1200 to 1800 UTC</td>
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<tr>
<td>Tendency (mb/6 h)</td>
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<td>ACY</td>
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<tr>
<th>3-h p Tendency and $\omega$ on 288 K Surface</th>
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<tbody>
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<td>1800 to 2100 UTC</td>
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<tr>
<td>Tendency (mb/3 h)</td>
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<tr>
<td>ACY</td>
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<td>WAL</td>
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System-Relative Flow on Isentropic Surfaces

• A key assumption is that the weather system translates horizontally without change in shape or intensity (anticyclone or cyclone changes only slowly over synoptic time scales).

• Another key assumption is locality.

• We subtract the speed of the system, C, from the ground-relative wind, V. V – C then becomes the system-relative flow on the isentropic surface, i.e., it is the flow relative to the moving system and takes into account the change in the isentropic topography.

• C is computed by tracking a feature on the isentropic surface, e.g., a vorticity maximum or a short-wave trough axis.

• Under the steady state assumption: Streamlines of system-relative wind are identical to trajectories!

• System-relative flow helps to identify conveyor belts, compute isentropic S-R vertical motion, and even S-R moisture transport vectors.
System-Relative (SR) Wind

\[ \vec{A} - \vec{B} = \vec{C} \]
Vector Subtraction

\[ \vec{V} - \vec{C} = SR \text{ Wind Vector} \]
Streamlines on a 500 hPa isobaric surface at a trough moving from the west at the same speed as the wind in the axis of the trough

\[ C = \quad \rightarrow \]
\[ V = \quad \rightarrow \]
\[ V - C = \quad \rightarrow \]

Streamlines as in (a) but for flow relative to the motion of the trough (from the west). Subtracting westerly motion vector from both sides of the trough results in deformation pattern.

Bader et al. (1995, Images in Weather Forecasting)
294 K S-R streamlines/isobars: 20 UTC 24 Dec 2002

C = 243.1° at 13.6 m s⁻¹
System-Relative Flow: How can it be used?

Carlson (1991, Mid-Latitude Weather Systems) notes:

• Relative-wind analysis reveals the existence of sharply-defined boundaries, which differentiate air streams of vastly differing moisture contents. Air streams tend to contain relatively narrow ranges of $\theta$ and $\theta_w$ peculiar to the air stream’s origins.

• Relative-wind isentropic analyses for synoptic-scale weather systems tend to show well-defined air streams, which have been identified by names – warm conveyor belt, cold conveyor belt, and the dry conveyor belt.

• A conveyor belt consists of an ensemble of air parcels having nearly the same $\theta_w$ (or $\theta_e$) value, starting from a common initial location, traveling over synoptic-scale time periods (> 18 h). They can be several hundreds of km wide and several km deep.
Understanding System-Relative Motion

The system motion has an effect on the vertical motion field on an isentropic surface. This can be accounted for by using system-relative motion instead of ground-relative flow.
System-Relative Isentropic Vertical Motion

Defined as:

$$\omega \sim (V - C) \cdot \nabla P$$

Where $\omega$ = system-relative vertical motion in $\mu\text{bars sec}^{-1}$

$V$ = wind velocity on the isentropic surface

$C$ = system velocity, and

$\nabla P$ = pressure gradient on the isentropic surface

$C$ is computed by tracking the associated vorticity maximum on the isentropic surface over the last 6 or 12 hours (one possible method); another method would be to track the motion of a short-wave trough on the isentropic surface.
System-Relative Isentropic Vertical Motion

$$\omega \approx (\vec{V} - \vec{C}) \cdot \nabla P$$

Including $C$, the speed of the system, is important when:

* the system is moving quickly and/or

* a significant component of the system motion is across the isobars on an isentropic surface, e.g.,

if the system motion is from SW-NE and the isobars are oriented N-S with lower pressure to the west, subtracting $C$ from $V$ is equivalent to “adding” a NE wind, thereby increasing the isentropic upslope.
In regions of isentropic upglide, this system-relative motion vector, $C$, will enhance the uplift (since $C$ is subtracted from the Velocity vector),

When is $C$ important to use when computing isentropic omegas?
Computing Isentropic Omegas

Essentially there are three approaches to computing isentropic omegas:

- **Ground-Relative Method** \((V \cdot \nabla P)\) :
  - Okay for slow-moving systems \(\partial P / \partial t\) term is small
  - Assumes that the advection term dominates (not always a good assumption)

- **System-Relative Method** \((V-C) \cdot \nabla P\) :
  - Good for situations in which the system is not deepening or filling rapidly
  - Also useful when the time step between map times is large (e.g., greater than 3 hours)
  - S-R velocity vectors are useful in computing S-R MTVs

- **Brute-Force Computational Method** \(\partial P / \partial t + V \cdot \nabla P\) :
  - Best for situations in which the system is rapidly deepening or filling
  - Good approximation when data are available at 3 h or less interval, allowing for good estimation of local time tendency of pressure
Mass Continuity Equation in Isentropic Coordinates

\[
\frac{\partial}{\partial t} \left( \frac{\partial P}{\partial \theta} \right) = -\hat{V} \cdot \nabla \left( \frac{\partial P}{\partial \theta} \right) - \frac{\partial P}{\partial \theta} (\nabla \cdot \hat{V}) - \frac{d \theta}{dt} \frac{\partial}{\partial \theta} \left( \frac{\partial P}{\partial \theta} \right) - \frac{\partial P}{\partial \theta} \frac{\partial}{\partial \theta} \left( \frac{d \theta}{dt} \right)
\]

**Term A:** Horizontal advection of static stability

**Term B:** Divergence/convergence changes the static stability; divergence (convergence) increases (decreases) the static stability

**Term C:** Vertical advection of static stability (via diabatic heating/cooling)

**Term D:** Vertical variation in the diabatic heating/cooling changes the static stability (e.g., decreasing (increasing) diabatic heating with height decreases (increases) the static stability
Term A: Horizontal Advection of Static Stability

- Very stable (50 hPa/4K)
- Less stable (100 hPa/4K)

Term B: Divergence/Convergence Effects

- Divergence

Term C: Vertical Advection of Static Stability

- Latent Heating

Term D: Vertical Variation of Diabatic Heating/Cooling

- Evaporative Cooling
- Latent Heating
What is a Conveyor Belt?

• A conveyor belt:
  • is an ensemble of air parcels having nearly the same $\theta_w$ (or $\theta_e$) value
  • originates from a common initial location
  • travels over synoptic-scale time periods (> 18 h).
  • is several hundreds of km wide and several km deep.
• System-relative wind and trajectory analysis reveals the existence of sharply-defined boundaries, which differentiate air streams of vastly differing moisture contents containing relatively narrow ranges of $\theta$ and $\theta_w$ peculiar to the air stream’s origins.
• Researchers have identified a warm conveyor belt, cold conveyor belt, and the dry conveyor belt.
Three-dimensional representation of the airflow in a developing mid-latitude cyclone (Palmen and Newton, 1969).
Relative trajectories of extruded stratospheric air during tropopause folding (full streamlines within plane of isentropic surface are reaching broken curve). Thin full curves denote surface isobars and the symbols L and H show the location of the surface or 500 mb cyclone or anticyclone centers. (Based on a figure by Danielsen, 1964.)
Fig. 6.17. Schematic illustration of the vertical displacement of a parcel trajectory approaching the cyclone from the south-southeast and passing through the area of precipitation associated with the cyclone and crossing the geopotential contours at the 700-mb level (marked by an open wave) at a significant angle.
Major Conveyor Belts

- Carlson (1980) noted that within an ETC there are three major conveyor belts:
  - Warm conveyor belt (WCB)
  - Cold conveyor belt (CCB)
  - Dry air stream or conveyor belt (DCB)
- Schultz (2001) proved that the CCB remained in the lower portion of the troposphere and turns cyclonically around the low.
- Martin (1999a) showed that the WCB will either turn anticyclonically or remain cyclonic around a low depending on the mid-level wind field with respect to the ETC lifecycle.
  - Called the cyclonically turning portion of the WCB as the “trowal airstream.”
  - Observed a spatial correlation between the trowal airstream and the trowal structure.

Conceptual model of airflow through an extratropical cyclone depicting the different conveyor belts from Carlson (1980) and the new revamped conveyor belt model overlaying on top (Schultz 2001).
Adapted from Dusan Djuric (1994) “Weather Analysis”, Chap. 10
Revised View of Conveyor Belts Illustrating 3-D Deformation

Ng (2005) Doctoral Dissertation